AHRQ Quality Indicators™

TESTING REPORT: THE MARKOV CHAIN MONTE CARLO INTEGRATION APPROXIMATION IN THE AHRQ QI PREDICTION MODULE CLOSELY MATCHES EXACT INTEGRATION RESULTS

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Introduction

The purpose of the Prediction Module (PM) is to predict functions of the outcome of interest while accounting for missing covariate values. In a regression model with no missing data, prediction is carried out by inserting the covariate values of a new observation into the estimated regression equation. When the covariates are missing, this direct calculation cannot be carried out. The PM reports the expected predicted value found by integrating over the distribution of the missing covariate values.

When the number of missing covariates is small, the integration can be accomplished directly by iterating over all possible combinations of missing covariates; the predicted value can be calculated exactly and quickly. When the number of missing covariates is large, integrating over the distribution of the missing covariate values is not computationally feasible; the calculations are straightforward but because of the large number of possible combinations of missing covariates, the calculations would take days and days on a normal computer. To achieve results in a reasonable period of time, (minutes or hours vs. days or weeks) the PM uses a well-established approximation to exact integration known as Markov Chain Monte Carlo (MCMC) integration. MCMC works by simulating draws from the distribution of the missing covariates and averaging the predicted values associated with these draws to estimate the expected value. It can be shown that the arithmetic average of the simulated draws approaches the desired expected predicted values as the number of simulated draws increases. See Givens and Hoeting (2005) and Robert and Casella (2004) for further discussion on MCMC methods and their properties. Because MCMC integration uses a randomly selected subset of all the possible combinations of missing values, two runs of the algorithm with identical input data will produce close but different results unless the random number generator's seed is set to the same value prior to conducting each run. Although a small degree of precision is lost when MCMC integration is used in place of exact integration, the predicted values obtained from MCMC can be obtained in far less time than the predicted values obtained from the exact method.

Battelle tested whether the MCMC integration performed by the PM was implemented as stated in the technical documentation (Morara, 2010) and whether the expected values calculated by the PM are close approximations of the exact values obtained using direct integration. We programmed a second implementation of the prediction module in the R statistical computing language (denoted PM-R) in which exact integration was used on problems with small numbers of covariates to calculate the expected predicted values. The person who programmed PM-R was not involved in the programming of the PM in C++ and relied entirely on the technical documentation to recreate the PM output. The choice of this testing model serves two purposes. First, by using exact integration, we can test whether the predicted values approximated by the PM using MCMC integration are close to the exact answers. Second, if the PM-R results are close to the PM results, (i.e., agree exactly when no covariates are missing and are within a few percent when data are missing) then we can be assured that the PM is performing its calculations correctly because the chance of two independent programmers making the same mistake is presumably quite small.

Test Inputs

The PM requires as inputs (1) a set of regression coefficients needed for prediction, and (2) a dataset for which predicted values are computed for each observation. A file in XML format specifies the file locations of the regression coefficients and the dataset.

An arbitrary predetermined set of coefficients was created and used to simulate a dataset. This simulated dataset was analyzed with the Analysis Module, resulting in a set of estimated coefficients. Two test samples with complete data were then simulated using the true coefficients. The test samples consist of 22 variables each: an outcome variable Y, a present on admission variable P, and covariate vectors $\boldsymbol{X}$ and $\boldsymbol{Z}$ each containing 10 variables. The Y and $\boldsymbol{Z}$ variables are never missing, but the data can have missing values of P and/or elements of $\boldsymbol{X}$. Values in the test samples were systematically deleted, starting with a random element of $\boldsymbol{X}$, to test the PM over various patterns of missing data. The testing of the PM utilized a total of 44 test observations: 2 test samples, each with 22 patterns of missing data. The test observations are listed in Table 1.

The 44 test observations were analyzed using both the PM and the PM-R. For each test case, the estimated regression coefficients were used in the prediction calculations. It is unlikely that a single run of the PM will produce a predicted value that matches the deterministic PM-R value exactly. Thus, each of the 44 test observations was run through the PM 1,000 times to obtain a distribution of the predicted values computed by the PM.

Note: The 1,000 repetitions of the PM on each test observation should not be confused with the 1,500 simulated draws used with *each run of the PM* to accomplish the MCMC. Each run of the MCMC uses a large user-selectable number of draws from the distribution of missing covariate patterns to estimate the predicted value. Each test observation was run through the PM 1,000 times, and each of those times the MCMC used 1,500 plausible combinations of covariates to calculate the predicted expected value of the integration.

Test Outputs

The PM produces the following seven output values:

1. Expected value of *Y* given ***Z***, denoted E[Y | Z]. Since *Y* and ***Z*** are always present in the data, this value should be identical in both PM and PM-R

2. Expected value of *Y* given ***X***, denoted E[Y | X].

3. Expected value of *Y* given that *P* is zero, denoted E[Y | P = 0]. This value is calculated as the product of *Y* and the probability that *P* equals zero. When *P* is non-missing,

$$\Pr(P = 0) = \begin{cases} 1 \text{ if observed } P = 0 \\ 0 \text{ if observed } P = 1 \end{cases}$$

Thus, when *P* is non-missing, E[Y | P = 0] equals 1 only if *Y* equals 1 and *P* equals zero; otherwise, E[Y | P = 0] is zero.

4. Expected value of *Y* given the covariate vector ***X*** and *P* is zero, denoted E[Y | X, P = 0]. This value equals zero whenever *P* is observed to equal 1.

5. Expected value of *Y* given the covariate vector ***X*** and *P* is zero using MCMC coefficients, denoted E[Y | X, P = 0]-MCMC. This value equals zero whenever *P* is observed to equal 1.

6. Expected value of *P* given the covariate vector ***X***, denoted E[P | X].

7. Expected value of *P* given *Y*, observed *P*, and the covariate vectors ***X*** and ***Z***, denoted E[P | YPXZ]. This value equals the probability that *P* equals 1. When *P* is non-missing, this value is simply the observed value of *P* – when *P* is observed to be zero, the probability that *P* equals 1 is zero. When *P* is missing, this value equals zero whenever *Y* equals zero – it is contradictory to have *P* equal to 1 yet not have the outcome present.

The above outputs, except for the first output, E[Y | Z], require integration when a data value needed for the calculation is missing. For example, E[Y | X] is calculated based on the ***X*** covariate vector. If one or more components of this vector are missing, then the PM will use MCMC integration to calculate the answer and the PM-R will use exact integration.

In some cases, a data value may be missing but integration is not needed because the calculation can be performed without the missing value. For example, the calculation of E[Y | X] does not depend on *P*, so the value of E[Y | X] should be the same regardless of whether *P* is missing. When *Y* equals zero, the only logical value for *P* is zero even if *P* is missing in the dataset.

When integration is not needed because the value can be calculated without regard to a missing element of the data, the values calculated by the PM and PM-R should match exactly. Table 2 describes the test observations that do not require integration to calculate the outputs.

Test Results

For each test observation in Table 1, the seven outputs were calculated using the PM-R. Each test observation was also analyzed 1,000 times using the PM, with each analysis using a different seed for the random number generator.

The only difference between the PM-R and the PM calculation is the method for integrating over the missing data: the PM-R uses exact integration while the PM uses MCMC integration. Therefore, we expect that in cases where no integration is necessary, all 1,000 runs of the PM will match the PM-R result exactly.

In cases where integration is necessary, the PM-R result should fall within the minimum and maximum of the 1,000 PM estimates. The PM-R result should fall close to the average of the 1,000 PM estimates; this would indicate that the MCMC integration produces an unbiased estimator of the result using exact integration. Ideally, the PM-R result would fall somewhere in the middle of the 1,000 PM estimates, with about 50% of the PM estimates less than the PM-R result and 50% of the PM estimates greater than the PM-R result. This would indicate that the distribution of the estimator based on MCMC integration is symmetric about the exact integration result.

Tables 3-9 report the results of the testing for each of the seven outputs. Within each table, the following values are listed for each of the test observations:

- The exact value of the output, as calculated by the PM-R
- The average of the 1,000 values calculated by the PM
- The minimum and maximum of the 1,000 values calculated by the PM
- The percentile of the PM-R value in relation to the 1,000 PM values (i.e., the percent of PM values that were less than the PM-R value)

In cases where no integration was performed, the percentile is not meaningful since all 1,000 runs of the PM returned the same value. The percentile in these cases is denoted with an asterisk.

The results in Table 3-9 clearly indicate that the PM is performing as expected. In each case where integration is not necessary, the PM returns the exact same value across all runs, and this value matches the calculation programmed in PM-R. When integration is necessary, the PM returns values that are centered around the exact PM-R value and the average PM value falls near the exact value. This can be seen by noting that the reported percentile values are all close to 50 percent. Aggregating across tables 3-9, nearly all (157 of 164) values in the column labeled 'Percentile' fall between 46% and 54%. The largest difference between the average PM value and the exact PM-R value occurred in the results for E[Y | X, P = 0] (see Table 6), Test Sample 2, missing data pattern 21. (PM-R = 0.000124 vs. average PM value of 0.000122) The difference is 0.000002, or 2% of the PM-R value. Again, the average difference is a function of the number of simulated draws per PM run (1,500 here) and could be made as small as is wished by trading off run-time for precision by conducting more and more draws.

References

Givens, G.H. and Hoeting, J.A. (2005). *Computational Statistics*. Wiley, Hoboken, NJ.

Robert, C.P. and Casella, G. (2004). *Monte Carlo Statistical Methods*, 2nd Ed. Springer, New York, NY.

Morara, Michele (2010), Internal Battelle Memorandum: Technical Notes on AHRQ Analysis, Prediction and Validation, March 2, 2010.

Table 1. Complete List of Test Observations

Missing data pattern	y	p	z.1	z.2	z.3	z.4	z.5	z.6	z.7	z.8	z.9	z.10	x.1	x.2	x.3	x.4	x.5	x.6	x.7	x.8	x.9	x.10
Test Sample 1																						
1	0	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0	1	1	0	1	1	1
2	0	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0		1	0	1	1	1
3	0	0	0	0	0	0	1	1	0	1	1	1	0	0		0		1	0	1	1	1
4	0	0	0	0	0	0	1	1	0	1	1	1	0	0		0		1	0	1		1
5	0	0	0	0	0	0	1	1	0	1	1	1	0	0		0		1	0			1
6	0	0	0	0	0	0	1	1	0	1	1	1	0	0		0		1	0			
7	0	0	0	0	0	0	1	1	0	1	1	1	0			0		1	0			
8	0	0	0	0	0	0	1	1	0	1	1	1	0			0		1				
9	0	0	0	0	0	0	1	1	0	1	1	1	0			0						
10	0	0	0	0	0	0	1	1	0	1	1	1	0									
11	0	0	0	0	0	0	1	1	0	1	1	1										
12	0		0	0	0	0	1	1	0	1	1	1	0	0	0	0	1	1	0	1	1	1
13	0		0	0	0	0	1	1	0	1	1	1	0	0	0	0		1	0	1	1	1
14	0		0	0	0	0	1	1	0	1	1	1	0	0		0		1	0	1	1	1
15	0		0	0	0	0	1	1	0	1	1	1	0	0		0		1	0	1		1
16	0		0	0	0	0	1	1	0	1	1	1	0	0		0		1	0			1
17	0		0	0	0	0	1	1	0	1	1	1	0	0		0		1	0			
18	0		0	0	0	0	1	1	0	1	1	1	0			0		1	0			
19	0		0	0	0	0	1	1	0	1	1	1	0			0		1				
20	0		0	0	0	0	1	1	0	1	1	1	0			0						
21	0		0	0	0	0	1	1	0	1	1	1	0									
22	0		0	0	0	0	1	1	0	1	1	1										
Test Sample 2																						
1	1	1	0	0	1	0	1	0	0	0	1	1	0	0	1	0	1	0	0	0	1	1
2	1	1	0	0	1	0	1	0	0	0	1	1	0	0		0	1	0	0	0	1	1
3	1	1	0	0	1	0	1	0	0	0	1	1	0	0		0	1	0	0		1	1
4	1	1	0	0	1	0	1	0	0	0	1	1	0	0			1	0	0		1	1
5	1	1	0	0	1	0	1	0	0	0	1	1	0	0			1	0	0			1
6	1	1	0	0	1	0	1	0	0	0	1	1	0	0			1		0			1
7	1	1	0	0	1	0	1	0	0	0	1	1	0	0			1					1

Missing data pattern	y	p	z.1	z.2	z.3	z.4	z.5	z.6	z.7	z.8	z.9	z.10	x.1	x.2	x.3	x.4	x.5	x.6	x.7	x.8	x.9	x.10
8	1	1	0	0	1	0	1	0	0	0	1	1	0	0			1					
9	1	1	0	0	1	0	1	0	0	0	1	1	0				1					
10	1	1	0	0	1	0	1	0	0	0	1	1					1					
11	1	1	0	0	1	0	1	0	0	0	1	1										
12	1		0	0	1	0	1	0	0	0	1	1	0	0	1	0	1	0	0	0	1	1
13	1		0	0	1	0	1	0	0	0	1	1	0	0		0	1	0	0	0	1	1
14	1		0	0	1	0	1	0	0	0	1	1	0	0		0	1	0	0		1	1
15	1		0	0	1	0	1	0	0	0	1	1	0	0			1	0	0		1	1
16	1		0	0	1	0	1	0	0	0	1	1	0	0			1	0	0			1
17	1		0	0	1	0	1	0	0	0	1	1	0	0			1		0			1
18	1		0	0	1	0	1	0	0	0	1	1	0	0			1					1
19	1		0	0	1	0	1	0	0	0	1	1	0	0			1					
20	1		0	0	1	0	1	0	0	0	1	1	0				1					
21	1		0	0	1	0	1	0	0	0	1	1					1					
22	1		0	0	1	0	1	0	0	0	1	1										

Table 2. Description of Test Observations Not Requiring Integration

<table>
<tr><th>Output</th><th>Test Observations NOT Requiring Integration</th><th>Reason</th></tr>
<tr><td>E[Y | Z]</td><td>All</td><td>No missing Y or Z values</td></tr>
<tr><td>E[Y | X]</td><td>Test patterns 1 and 12 for both samples</td><td>No missing X values</td></tr>
<tr><td rowspan="2">E[Y | P = 0]</td><td>Test patterns 1-11 for both samples</td><td>No missing P values</td></tr>
<tr><td>Test patterns 12-22 for Test Sample 1</td><td>Y = 0, so missing P value must be zero</td></tr>
<tr><td rowspan="3">E[Y | X, P = 0]</td><td>Test pattern 1 for Test Sample 1</td><td>No missing P or X values</td></tr>
<tr><td>Test pattern 12 for Test Sample 1</td><td>No missing X values; Y = 0, so missing P value must be zero</td></tr>
<tr><td>Test patterns 1-11 for Test Sample 2</td><td>Observed P value equals 1, so result is always zero.</td></tr>
<tr><td rowspan="3">E[Y | X, P = 0]-MCMC</td><td>Test pattern 1 for Test Sample 1</td><td>No missing P or X values</td></tr>
<tr><td>Test pattern 12 for Test Sample 1</td><td>No missing X values; Y = 0, so missing P value must be zero</td></tr>
<tr><td>Test patterns 1-11 for Test Sample 2</td><td>Observed P value equals 1, so result is always zero.</td></tr>
<tr><td>E[P | X]</td><td>Test patterns 1 and 12 for both samples</td><td>No missing X values</td></tr>
<tr><td rowspan="2">E[P | YPXZ]</td><td>Test patterns 1-11 for both samples</td><td>No missing P values</td></tr>
<tr><td>Test patterns 12-22 for Test Sample 1</td><td>Y = 0, so missing P value must be zero</td></tr>
</table>

Table 3. Test Results for E[Y | Z]

Missing data pattern	PM-R	Average PM	Min PM	Max PM	Percentile
Test Sample 1					
1	0.044555	0.044555	0.044555	0.044555	*
2	0.044555	0.044555	0.044555	0.044555	*
3	0.044555	0.044555	0.044555	0.044555	*
4	0.044555	0.044555	0.044555	0.044555	*
5	0.044555	0.044555	0.044555	0.044555	*
6	0.044555	0.044555	0.044555	0.044555	*
7	0.044555	0.044555	0.044555	0.044555	*
8	0.044555	0.044555	0.044555	0.044555	*
9	0.044555	0.044555	0.044555	0.044555	*
10	0.044555	0.044555	0.044555	0.044555	*
11	0.044555	0.044555	0.044555	0.044555	*
12	0.044555	0.044555	0.044555	0.044555	*
13	0.044555	0.044555	0.044555	0.044555	*
14	0.044555	0.044555	0.044555	0.044555	*
15	0.044555	0.044555	0.044555	0.044555	*
16	0.044555	0.044555	0.044555	0.044555	*
17	0.044555	0.044555	0.044555	0.044555	*
18	0.044555	0.044555	0.044555	0.044555	*
19	0.044555	0.044555	0.044555	0.044555	*
20	0.044555	0.044555	0.044555	0.044555	*
21	0.044555	0.044555	0.044555	0.044555	*
22	0.044555	0.044555	0.044555	0.044555	*
Test Sample 2					
1	0.202022	0.202022	0.202022	0.202022	*
2	0.202022	0.202022	0.202022	0.202022	*
3	0.202022	0.202022	0.202022	0.202022	*
4	0.202022	0.202022	0.202022	0.202022	*
5	0.202022	0.202022	0.202022	0.202022	*
6	0.202022	0.202022	0.202022	0.202022	*
7	0.202022	0.202022	0.202022	0.202022	*
8	0.202022	0.202022	0.202022	0.202022	*
9	0.202022	0.202022	0.202022	0.202022	*
10	0.202022	0.202022	0.202022	0.202022	*
11	0.202022	0.202022	0.202022	0.202022	*
12	0.202022	0.202022	0.202022	0.202022	*
13	0.202022	0.202022	0.202022	0.202022	*
14	0.202022	0.202022	0.202022	0.202022	*
15	0.202022	0.202022	0.202022	0.202022	*
16	0.202022	0.202022	0.202022	0.202022	*
17	0.202022	0.202022	0.202022	0.202022	*
18	0.202022	0.202022	0.202022	0.202022	*
19	0.202022	0.202022	0.202022	0.202022	*
20	0.202022	0.202022	0.202022	0.202022	*
21	0.202022	0.202022	0.202022	0.202022	*
22	0.202022	0.202022	0.202022	0.202022	*

* Percentile not meaningful because all 1,000 runs returned the same result.

Table 4. Test Results for E[Y | X]

Missing data pattern	PM-R	Average PM	Min PM	Max PM	Percentile
Test Sample 1					
1	0.032978	0.032978	0.032978	0.032978	*
2	0.031499	0.031489	0.030926	0.031972	52.2
3	0.033116	0.03312	0.031996	0.033925	48.6
4	0.032995	0.032988	0.032159	0.033893	51
5	0.040191	0.040187	0.036293	0.044373	53.5
6	0.040367	0.040316	0.037241	0.044033	53.3
7	0.040945	0.04092	0.037601	0.044889	52.1
8	0.041781	0.041739	0.038454	0.045084	50.5
9	0.040799	0.040794	0.037749	0.044355	51.5
10	0.042111	0.042103	0.03877	0.046389	50.5
11	0.041929	0.041929	0.038507	0.045399	50.4
12	0.032978	0.032978	0.032978	0.032978	*
13	0.031499	0.031497	0.030986	0.031972	49
14	0.033116	0.033103	0.032205	0.034027	52
15	0.032995	0.032999	0.032174	0.033793	49.7
16	0.040191	0.040177	0.037523	0.043409	52.4
17	0.040367	0.040352	0.036996	0.044121	52.1
18	0.040945	0.040886	0.037941	0.044212	52.9
19	0.041781	0.04173	0.038831	0.045523	52.9
20	0.040799	0.04084	0.037749	0.044355	49.8
21	0.042111	0.042088	0.037774	0.045415	51.4
22	0.041929	0.041898	0.038458	0.046351	52.8
Test Sample 2					
1	0.17838	0.17838	0.17838	0.17838	*
2	0.176941	0.176938	0.175877	0.177865	49.7
3	0.174381	0.174385	0.172024	0.176417	48.2
4	0.18217	0.18216	0.178232	0.186147	50.9
5	0.181825	0.18183	0.177773	0.185839	50.8
6	0.187906	0.187929	0.183901	0.192441	48.4
7	0.191589	0.191566	0.186885	0.196067	51.7
8	0.192226	0.192111	0.187136	0.196636	53.1
9	0.196348	0.196346	0.191423	0.202941	50.9
10	0.195601	0.195586	0.190469	0.201249	50
11	0.192604	0.192467	0.187176	0.198235	54.8
12	0.17838	0.17838	0.17838	0.17838	*
13	0.176919	0.176908	0.175877	0.177718	52
14	0.174308	0.174325	0.171235	0.17627	48
15	0.182407	0.182383	0.17771	0.187022	50.9
16	0.182037	0.182094	0.178516	0.186926	49.7
17	0.188032	0.18801	0.183247	0.193926	50.5
18	0.191724	0.191701	0.187192	0.196122	51.4
19	0.192379	0.192419	0.18786	0.197583	48.7
20	0.196371	0.196355	0.191132	0.201307	50.4
21	0.195649	0.195598	0.190814	0.202453	51.8
22	0.192364	0.192337	0.187332	0.19846	51.4

* Percentile not meaningful because all 1,000 runs returned the same result.

Table 5. Test Results for E[Y | P = 0]

Missing data pattern	PM-R	Average PM	Min PM	Max PM	Percentile
Test Sample 1					
1	0	0	0	0	*
2	0	0	0	0	*
3	0	0	0	0	*
4	0	0	0	0	*
5	0	0	0	0	*
6	0	0	0	0	*
7	0	0	0	0	*
8	0	0	0	0	*
9	0	0	0	0	*
10	0	0	0	0	*
11	0	0	0	0	*
12	0	0	0	0	*
13	0	0	0	0	*
14	0	0	0	0	*
15	0	0	0	0	*
16	0	0	0	0	*
17	0	0	0	0	*
18	0	0	0	0	*
19	0	0	0	0	*
20	0	0	0	0	*
21	0	0	0	0	*
22	0	0	0	0	*
Test Sample 2					
1	0	0	0	0	*
2	0	0	0	0	*
3	0	0	0	0	*
4	0	0	0	0	*
5	0	0	0	0	*
6	0	0	0	0	*
7	0	0	0	0	*
8	0	0	0	0	*
9	0	0	0	0	*
10	0	0	0	0	*
11	0	0	0	0	*
12	0.039096	0.039077	0.02	0.06	53.4
13	0.039378	0.039453	0.019	0.061	50.9
14	0.039732	0.039733	0.023	0.064	47.9
15	0.042076	0.04212	0.024	0.063	53
16	0.042572	0.042587	0.025	0.065	51.8
17	0.041682	0.04196	0.021	0.066	49.1
18	0.041954	0.042172	0.022	0.062	46.3
19	0.043461	0.043445	0.025	0.065	51.7
20	0.042338	0.042401	0.023	0.061	51.4
21	0.04159	0.041227	0.023	0.068	52.9
22	0.044233	0.04431	0.025	0.065	53.5

* Percentile not meaningful because all 1,000 runs returned the same result.

Table 6. Test Results for E[Y | X, P = 0]

Missing data pattern	PM-R	Average PM	Min PM	Max PM	Percentile
Test Sample 1					
1	0.000512	0.000512	0.000512	0.000512	*
2	0.000526	0.000526	0.000521	0.000531	47.8
3	0.00053	0.00053	0.000525	0.000535	50.5
4	0.000533	0.000533	0.000527	0.00054	51.4
5	0.0006	0.0006	0.000572	0.00064	53
6	0.00076	0.000759	0.00069	0.000829	51.6
7	0.000754	0.000754	0.000693	0.000818	51
8	0.0008	0.000799	0.000711	0.000877	52.5
9	0.00079	0.000789	0.000718	0.000875	51.1
10	0.000879	0.000878	0.00079	0.001004	53.4
11	0.000862	0.000863	0.000763	0.000993	50.9
12	0.000512	0.000512	0.000512	0.000512	*
13	0.000526	0.000526	0.000521	0.00053	51
14	0.00053	0.00053	0.000525	0.000535	51
15	0.000533	0.000533	0.000527	0.000539	48.3
16	0.0006	0.0006	0.000572	0.000632	53.8
17	0.00076	0.000759	0.000687	0.000835	50.8
18	0.000754	0.000753	0.000691	0.00082	51.2
19	0.0008	0.000799	0.000726	0.000878	51
20	0.00079	0.00079	0.00072	0.000886	50.8
21	0.000879	0.000878	0.000784	0.001015	52.4
22	0.000862	0.000862	0.000747	0.000993	53.7
Test Sample 2					
1	0	0	0	0	*
2	0	0	0	0	*
3	0	0	0	0	*
4	0	0	0	0	*
5	0	0	0	0	*
6	0	0	0	0	*
7	0	0	0	0	*
8	0	0	0	0	*
9	0	0	0	0	*
10	0	0	0	0	*
11	0	0	0	0	*
12	0.000058	0.000058	0.00003	0.000089	53.4
13	0.000058	0.000059	0.000028	0.000091	50.9
14	0.000058	0.000058	0.000033	0.000093	49.1
15	0.000085	0.000085	0.000039	0.000138	53.8
16	0.000087	0.000086	0.000043	0.000157	55.1
17	0.000086	0.000086	0.000045	0.000155	51.7
18	0.000093	0.000093	0.000039	0.000155	51.8
19	0.000131	0.000131	0.000055	0.000272	54.5
20	0.000127	0.000127	0.000043	0.000245	54
21	0.000124	0.000122	0.000044	0.000251	57.9
22	0.000137	0.000138	0.000066	0.000287	53.2

* Percentile not meaningful because all 1,000 runs returned the same result.

Table 7. Test Results for E[Y | X, P = 0]-MCMC

Missing data pattern	PM-R	Average PM	Min PM	Max PM	Percentile
Test Sample 1					
1	0.001952	0.001952	0.001952	0.001952	*
2	0.001962	0.001962	0.001959	0.001966	47.8
3	0.002021	0.002021	0.00199	0.002043	49.3
4	0.002034	0.002034	0.002011	0.002071	49.3
5	0.002392	0.002391	0.002249	0.002606	53.7
6	0.002461	0.002458	0.002306	0.002652	52.5
7	0.002442	0.002441	0.00228	0.002639	52.5
8	0.002498	0.002496	0.002321	0.002654	52.2
9	0.002471	0.002471	0.002339	0.002653	50
10	0.002725	0.002721	0.002522	0.002999	52.5
11	0.002674	0.002673	0.002422	0.002967	51.7
12	0.001952	0.001952	0.001952	0.001952	*
13	0.001962	0.001962	0.001959	0.001965	51
14	0.002021	0.00202	0.001998	0.002043	54
15	0.002034	0.002034	0.002008	0.002057	48.5
16	0.002392	0.002391	0.002247	0.002573	53.6
17	0.002461	0.00246	0.002287	0.002667	52.4
18	0.002442	0.002438	0.002293	0.002601	53.5
19	0.002498	0.002496	0.002358	0.002692	51.8
20	0.002471	0.002472	0.002328	0.002635	48.4
21	0.002725	0.002722	0.002504	0.002973	51.1
22	0.002674	0.002673	0.002444	0.002947	54.4
Test Sample 2					
1	0	0	0	0	*
2	0	0	0	0	*
3	0	0	0	0	*
4	0	0	0	0	*
5	0	0	0	0	*
6	0	0	0	0	*
7	0	0	0	0	*
8	0	0	0	0	*
9	0	0	0	0	*
10	0	0	0	0	*
11	0	0	0	0	*
12	0.000385	0.000385	0.000197	0.000591	53.4
13	0.000385	0.000385	0.000187	0.000601	50.9
14	0.00038	0.00038	0.000219	0.000609	49.4
15	0.000541	0.000539	0.000253	0.00087	52.9
16	0.000554	0.000551	0.00028	0.000994	54.7
17	0.000547	0.000547	0.000291	0.000975	51.3
18	0.000565	0.000568	0.000242	0.000964	52
19	0.000608	0.000607	0.000301	0.001048	53.2
20	0.00059	0.000593	0.000248	0.001025	51
21	0.000578	0.000572	0.000229	0.001127	55.2
22	0.00062	0.000624	0.000298	0.000975	51.6

* Percentile not meaningful because all 1,000 runs returned the same result.

Table 8. Test Results for E[P | X]

Missing data pattern	PM-R	Average PM	Min PM	Max PM	Percentile
Test Sample 1					
1	0.033061	0.033061	0.033061	0.033061	*
2	0.031411	0.031399	0.03077	0.031938	52.2
3	0.033343	0.033348	0.032031	0.0343	48.3
4	0.03318	0.033171	0.032198	0.034237	51.4
5	0.041011	0.041007	0.036656	0.045617	53.2
6	0.040708	0.040653	0.037251	0.044622	53.7
7	0.041564	0.041537	0.037799	0.045751	52.8
8	0.042143	0.042096	0.038523	0.046003	50.7
9	0.041064	0.04106	0.03765	0.045123	50.1
10	0.042137	0.042133	0.038391	0.04682	50.9
11	0.042139	0.042135	0.038354	0.046042	50.1
12	0.033061	0.033061	0.033061	0.033061	*
13	0.031411	0.031408	0.030838	0.031938	49
14	0.033343	0.033328	0.032275	0.034415	51.6
15	0.03318	0.033185	0.032224	0.034102	49.1
16	0.041011	0.040996	0.038051	0.044538	52.9
17	0.040708	0.040693	0.037084	0.04464	52.4
18	0.041564	0.041507	0.038255	0.045211	52.6
19	0.042143	0.042085	0.038875	0.045956	52.6
20	0.041064	0.04111	0.03765	0.045086	50.2
21	0.042137	0.042116	0.037307	0.045969	51.1
22	0.042139	0.042106	0.038598	0.046964	52
Test Sample 2					
1	0.19498	0.19498	0.19498	0.19498	*
2	0.193251	0.193248	0.191974	0.194362	49.7
3	0.190412	0.190416	0.187738	0.192701	49.3
4	0.197361	0.197356	0.193569	0.201094	51.1
5	0.196852	0.196853	0.193003	0.200562	50.5
6	0.204187	0.204207	0.200093	0.20889	48.2
7	0.206927	0.20689	0.201946	0.211505	51.7
8	0.205715	0.205591	0.200802	0.210185	53.3
9	0.211742	0.211763	0.206636	0.218286	51.6
10	0.211746	0.211733	0.206203	0.217786	50.8
11	0.208106	0.207962	0.202521	0.213416	53.9
12	0.19498	0.19498	0.19498	0.19498	*
13	0.193226	0.193212	0.191974	0.194185	52
14	0.19033	0.190349	0.186855	0.192524	48.3
15	0.197553	0.197533	0.192791	0.202072	51
16	0.197005	0.197071	0.193549	0.202122	49
17	0.204237	0.204204	0.199125	0.209946	50.5
18	0.206983	0.206948	0.202104	0.211157	50.9
19	0.205747	0.205774	0.200813	0.210661	49.1
20	0.211587	0.211574	0.206275	0.217936	49.7
21	0.211593	0.21157	0.205782	0.21836	50.2
22	0.2076	0.207559	0.202107	0.214162	50.7

* Percentile not meaningful because all 1,000 runs returned the same result.

Table 9. Test Results for E[P | YPXZ]

Missing data pattern	PM-R	Average PM	Min PM	Max PM	Percentile
Test Sample 1					
1	0	0	0	0	*
2	0	0	0	0	*
3	0	0	0	0	*
4	0	0	0	0	*
5	0	0	0	0	*
6	0	0	0	0	*
7	0	0	0	0	*
8	0	0	0	0	*
9	0	0	0	0	*
10	0	0	0	0	*
11	0	0	0	0	*
12	0	0	0	0	*
13	0	0	0	0	*
14	0	0	0	0	*
15	0	0	0	0	*
16	0	0	0	0	*
17	0	0	0	0	*
18	0	0	0	0	*
19	0	0	0	0	*
20	0	0	0	0	*
21	0	0	0	0	*
22	0	0	0	0	*
Test Sample 2					
1	1	1	1	1	*
2	1	1	1	1	*
3	1	1	1	1	*
4	1	1	1	1	*
5	1	1	1	1	*
6	1	1	1	1	*
7	1	1	1	1	*
8	1	1	1	1	*
9	1	1	1	1	*
10	1	1	1	1	*
11	1	1	1	1	*
12	0.960904	0.960923	0.94	0.98	46.6
13	0.960622	0.960547	0.939	0.981	49.1
14	0.960268	0.960267	0.936	0.977	52.1
15	0.957924	0.95788	0.937	0.976	47
16	0.957428	0.957413	0.935	0.975	48.2
17	0.958318	0.95804	0.934	0.979	50.9
18	0.958046	0.957828	0.938	0.978	53.7
19	0.956539	0.956555	0.935	0.975	48.3
20	0.957662	0.957599	0.939	0.977	48.6
21	0.95841	0.958773	0.932	0.977	47.1
22	0.955767	0.95569	0.935	0.975	46.5

* Percentile not meaningful because all 1,000 runs returned the same result.