AHRQ Quality Indicators

Quality Indicator Empirical Methods

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Overview

This document describes the empirical methods used in the calculation of the AHRQ Quality Indicators (AHRQ QI). Topics covered include the creation of the analysis data; the flagging of discharges for inclusion and exclusion based on the indicator numerator and denominator specifications; the calculation of observed, expected, risk-adjusted and smoothed rates and confidence and probability intervals; and the calculation of composite values, weights, and probability intervals. The empirical methods described below are implemented in the AHRQ QI software to calculate rates based on the user's data and used in the calculation of comparative data embedded in the software (i.e. reference population rates and covariate coefficients); however, these empirical methods also guide the development of candidate AHRQ QIs.

Empirical Methods

State Inpatient Databases

The specifications for the AHRQ QIs are based on the data elements and data values of the State Inpatient Databases (SID), Healthcare Cost and Utilization Project (HCUP), Agency for Healthcare Research and Quality (AHRQ).

Table A.1 Required Data Elements from SID

Data Element	Label	PQI	IQI	PSI	PDI
AGE	Age in years at admission	X	X	X	X
AGEDAY	Age in days (when age < 1 year)				X
ASCHED	Admission scheduled vs. unscheduled			X	X
ASOURCE	Admission source (uniform)	X	X	X	X
ATYPE	Admission type			X	X
DISCWT	Weight to discharges in the universe (NIS Only)	X	X	X	X
DISPUNIFORM	Disposition of patient (uniform)		X	X	X
DQTR	Discharge quarter	X	X	X	X
DRG	DRG in effect on discharge date	X	X	X	X
DRGVER	DRG grouper version used on discharge date	X	X	X	X
DSHOSPID	Data source hospital identifier		X	X	X
DX1-DX30	Diagnosis	X	X	X	X
DXPOA1-DXPOA30	Diagnosis present on admission indicator		X	X	X
E_POA1-E_POA10	E code present on admission indicator		X	X	X
ECODE1-ECODE10	E code		X	X	X
FEMALE	Indicator of sex	X	X	X	X
HOSPST	Hospital state postal code		X	X	X
KEY	HCUP record identifier	X	X	X	X
LOS	Length of stay (cleaned)		X	X	X
MDC	MDC in effect on discharge date	X	X	X	X
PAY1	Primary expected payer (uniform)		X	X	X
PAY2	Secondary expected payer (uniform)		X	X	X
POINTOFORIGINUB04	Point of origin for admission or visit, UB-04 standard coding	X	X	X	X
PR1-PR30	Procedure	X	X	X	X
PRDAY1-PRDAY30	Number of days from admission			X	X

Data Element	Label	PQI	IQI	PSI	PDI
PSTCO	Patient state/county FIPS code	X	X	X	X
PSTCO2	Patient state/county FIPS code, possibly derived from ZIP Code	X	X	X	X
RACE	Race (uniform)	X	X	X	X
SEX	Sex	X	X	X	
YEAR	Calendar year	X	X	X	X

Source: http://www.hcup-us.ahrq.gov/databases.jsp

In the preparation of the analysis data used for the reference population rates and other parameters, data elements and data values shown in Table A.2 are constructed from the SID.

Table A.2 Data Elements and Data Values Constructed from the SID

SID		AHRQ QI	
Data Element	**Data Value**	**Data Element**	**Data Value**
FEMALE	0 – Male 1 – Female	SEX	1 – Male 2 – Female
ATYPE, ASCHED and AGEDAY	IF ATYPE = Missing AND ASCHED = 1 (Scheduled admission) AND AGEDAY ~= 0	ATYPE	3- Elective
ECODE1-ECODE10	As reported	DX31-DX40	As reported
E_POA1-E_POA10	As reported	DXPOA31-DXPOA40	As reported

Note: Missing values for FEMALE are assigned to SEX = 1 (Male) because in the SID the data element is missing in less than 1% of discharges. An alternative would be to impute SEX based on other data elements (e.g., diagnosis code). Note that in the AHRQ QI software discharges in the user's data with missing values for SEX are deleted.

The discharges in the SID are assigned to the adult or pediatric analysis data based on age and Major Diagnostic Category (MDC) (Table A.3). Discharges in MDC 14 (Pregnancy, Childbirth & the Puerperium) are assigned to the adult analysis data regardless of age.

Table A.3 Analysis data Inclusion Rule

Analysis data	Inclusion Rule
Adult	AGE greater than or equal to 18 or MDC equal to 14
Pediatric	AGE less than 18 and MDC not equal to 14

The adult analysis data are used as the reference population for the Prevention Quality Indicators (PQIs), the Inpatient Quality Indicators (IQIs), and the Patient Safety Indicators (PSIs). The pediatric analysis data are used as the reference population for the Pediatric Quality Indicators (PDIs), the Neonatal Quality Indicators (NQI) and indicators from other modules defined on pediatric discharges (i.e., PQI 09, PSI 17).

Beginning in Version 4.3, discharges from non-community hospitals are deleted from the adult and pediatric analysis data. Community hospitals, as defined by American Hospital Association (AHA), include "all nonfederal, short-term, general and other specialty hospitals, excluding hospital units of institutions." Included among community hospitals are academic medical

centers and specialty hospitals such as obstetrics, gynecology, ear nose throat, short-term rehabilitation, orthopedic, and pediatric hospitals. Non-community hospitals include federal hospitals (Veterans Administration, Department of Defense, and Indian Health Service hospitals), long-term hospitals, psychiatric hospitals, alcohol/chemical dependency treatment facilities and hospitals units within institutions such as prisons.
(See http://hcup-us.ahrq.gov/db/state/siddist/siddist_hospital.jsp#2008).

No other edits are applied to the State Inpatient Databases (SID). Additional information on the processing of the HCUP SID may be found on the HCUP web site.
(http://www.hcup-us.ahrq.gov/db/state/siddbdocumentation.jsp)

Discharge Level Flags

Discharges are flagged for inclusion or exclusion from the numerator and denominator for each AHRQ QI based on the data elements, data values, and logic described in the technical specifications for each module. For the technical specifications, see the following references.
(See http://www.qualityindicators.ahrq.gov/modules/pqi_resources.aspx)
(See http://www.qualityindicators.ahrq.gov/modules/iqi_resources.aspx)
(See http://www.qualityindicators.ahrq.gov/modules/psi_resources.aspx)
(See http://www.qualityindicators.ahrq.gov/modules/pdi_resources.aspx)

Present on Admission

Present on Admission (POA) was added as a data element to the UB-04 in fiscal year 2008 (effective March 1, 2007). The POA data element (Table B.1) applies to each principal and secondary diagnosis code and provides a means of distinguishing pre-existing co-morbidities from complications that occur during the hospitalization of interest. POA is defined as "present at the time the order for inpatient admission occurs."[1] Conditions that develop during an outpatient encounter, including treatment in an emergency department, are considered as present on admission. Current AHRQ QI that use POA are listed in Appendix A.

Several states have adopted POA in the discharge data submitted by hospitals to either the state department of health or the state hospital association. Twenty-two (22) states provided this data element to AHRQ for the 2008 HCUP SID.

Table B.1. Values for the Present on Admission Data Element

ICD-9-CM Guidelines	Description	AHRQ QI POA Data Element	Description
Y - Yes	Present at the time of inpatient admission	1	Diagnosis present at admission
N – No	Not present at the time of inpatient admission	0	Diagnosis not present at admission

[1] http://www.cdc.gov/nchs/data/icd9/icdguide10.pdf.

ICD-9-CM Guidelines	Description	AHRQ QI POA Data Element	Description
U - Unknown	Documentation is insufficient to determine if condition is present on admission	0	Diagnosis not present at admission
W – Clinically undetermined	Provider is unable to clinically determine whether condition was present on admission or not	1	Diagnosis present at admission
E - Unreported/Not used; Also includes UB-04 values of "1"	Exempt from POA reporting	1	Diagnosis present at admission

Source: http://www.cms.hhs.gov/HospitalAcqCond/05_Coding.asp#TopOfPage; http://www.hcup-us.ahrq.gov/db/vars/siddistnote.jsp?var=e_poan.

The empirical methods do not assume that every discharge record in the analysis data has POA data. POA may be available in some states and not other states, in some hospitals within states and not other hospitals, or in some discharge records within hospitals and not other discharge records.

For purposes of the AHRQ QI, the principal diagnosis is always assumed Present on Admission by definition, regardless of the coding of the POA data element in the principal field. Secondary diagnosis codes are considered present on admission if the POA data element is coded with a Y, W, E or 1. Secondary diagnosis codes are considered not present on admission if the POA data element is coded with a N, U or 0. When the POA data element is blank (missing), secondary diagnosis codes are considered present on admission for purposes of risk-adjustment and not present on admission for purposes of the outcome of interest.

Numerator

Discharges are flagged for inclusion in the numerator of each AHRQ QI according to the specification for the *outcome of interest*. Discharges flagged for inclusion in the numerator are assigned a value of “1” to the discharge level indicator data element.

For discharge records with POA data, the discharge is flagged to indicate whether the outcome of interest was present on admission by assigning a value of “1” to the discharge level POA exclusion data element. Otherwise a value of “0” is assigned to the discharge level POA exclusion data element. The outcome of interest is considered present on admission if any of the diagnosis codes that define the outcome of interest are coded as present on admission.

For discharge records without POA data, a value of “missing” is assigned to the POA exclusion data element.

Denominator

Discharges are flagged for inclusion in the denominator of each AHRQ QI according to the specification for the *population at risk*. Discharges flagged for inclusion in the denominator are

assigned a value of "0" in the discharge level indicator data element unless the discharge is also in the outcome of interest in which case the value of "1" is assigned. Discharges in the outcome of interest are in the population at risk by definition.

Exclusions

Numerator Exclusions. Discharges are flagged for exclusion from the numerator of an AHRQ QI if the outcome of interest has more than one component, and the discharge is not in the population at risk for one component but remains in the population at risk for another component. These discharges are assigned a value of "0" in the discharge level indicator data element.

Denominator Exclusions. Generally, discharges are flagged for exclusion from the denominator for an AHRQ QI if the outcome of interest is more likely than not to be present on admission, less likely than not to be preventable, or the probability of experiencing the outcome of interest is zero or almost zero. These discharges are assigned a value of "missing" in the discharge level indicator data element.

For those discharge records with POA data, the discharge is flagged to indicate whether the discharge record has an excluding condition. An excluding condition is defined as a condition where the outcome of interest, if present, is more likely to be present on admission or less likely to be preventable. The discharge is flagged by assigning a value of "1" to the discharge level POA exclusion data element. Otherwise a value of "0" is assigned to the discharge level POA exclusion data element (unless the POA exclusion data element was previously assigned a value of "1" because the outcome of interest was coded as present on admission).

For discharge records without POA data, a value of "missing" is assigned to the POA exclusion data element.

Covariates

Discharges are flagged for inclusion in the covariates based on the data elements, data values, and logic described in the technical specifications. For a given covariate, if the discharge meets the technical specification for that covariate a value of "1" is assigned to the discharge level covariate data element. Otherwise a value of "0" is assigned to the discharge level covariate data element.

For the PSIs, discharges are flagged to indicate whether the discharge record meets the technical specification for gender, age, modified Diagnosis-Related Group (MDRG) and at least one of twenty-five (25) co-morbidities that are used as covariates in the risk adjustment model. For the IQIs, discharges are flagged to indicate whether the discharge record meets the technical specification for gender, age, All Patient Refined Diagnosis Related Groups (APR-DRG) and risk-of-mortality subclass (minor, moderate, major, extreme) that are used as covariates in the risk adjustment model. For the PDI, discharges are flagged to indicate whether the discharge record meets the technical specification for birth weight, age in days, age in years, modified Diagnosis-Related Group (MDRG), at least one of forty-six (46) clinical classification software

(CCS) co-morbidities and some indicator-specific risk categories that are used as covariates in the risk adjustment model.

For discharge records with POA data, the software creates a second set of data elements (i.e., the **Z** data elements used in the modeling described below) that do not consider secondary diagnosis codes that are not present on admission when assigning comorbidity or risk-of-mortality flags.

For the PQIs, discharges are flagged to indicate whether the discharge record meets the technical specification for gender, age in 5-year groups and poverty category that are used as covariates in the risk adjustment model.

Rate Calculation

Selection of Covariates

For the provider level indicators, each module has a standard set of covariates grouped into four categories: demographics, severity of illness, comorbidities and other (See Appendix Table B.1). The standard set is then tailored to each indicator to create a parsimonious set of covariates for each indicator. Based on cross tabulations between each covariate and the outcome of interest, only those covariates with at least 30 cases with the outcome of interest are retained. For categories that are mutually exclusive, covariates with fewer than 30 cases are pooled into the next covariate along the risk gradient. For example, age 70 to 74 is combined with age 65 to 69, or risk of mortality subclass 3 is combined with subclass 2. For categories with no risk gradient, covariates are pooled into broader covariates. For example, MS-DRGs are pooled into MDCs.

The omitted covariate within mutually exclusive categories are 1) the most common and/or 2) the least risk. However, the selection of the omitted category does not impact model performance.

Once the preliminary model is specified, the model is estimated on the adult or pediatric analytic data, as appropriate. Only those covariates that are statistically significant ($p<.05$) are retained. For covariates that are not statistically significant in categories that are mutually exclusive, the pooling process described above is repeated until a complete, parsimonious model is specified.

For the area level indicators, the models use the complete set of covariates for gender, age in 5-year age groups, an interaction with gender * age. There is also an optional set of covariates for poverty category based on the county of patient residence.

Observed Rate

The formula for the observed rate is as follows:

$$Observed\ Rate\ = \frac{Count\ of\ discharges\ of\ the\ outcome\ of\ interest}{Count\ of\ discharges\ in\ the\ population\ at\ risk}$$

Expected Rate

The formula for the expected rate is as follows:

$$Expected\ rate\ = \frac{Sum\ of\ the\ predicted\ rate\ for\ each\ discharge}{Count\ of\ discharges\ in\ the\ population\ at\ risk}$$

Using a logistic regression we can model the predicted rate as

$$Predicted\ rate = \frac{\exp(\boldsymbol{X}_i\boldsymbol{\beta})}{1 + \exp(\boldsymbol{X}_i\boldsymbol{\beta})}$$

where $\boldsymbol{X}_i$ is an explanatory vector of 0 and/or 1 (covariate) containing information such as: demographics, severity of illness, co-morbidity and other; $\boldsymbol{\beta}$ is a vector of coefficients estimated from a binomial model on the adult or pediatric analytic data.

In the following sections we will give an improved formula for the predicted probability (equation (8), below), conditional to the POA data and summed over the values of the missing data.

Incorporating POA in the Expected Rate

The risk-adjustment model and calculation of the expected rate extends the general formula above in order to incorporate present on admission data. The general intent that informed the risk-adjustment model was to develop an approach that used all of the available data (with or without the POA data element) for calculating the comparative data and the observed, expected, risk-adjusted and smoothed rates. The approach was to be incremental in that the estimation would improve over time as additional states and payers adopted POA. From the perspective of individual hospitals, each hospital could decide whether or not it was worth the additional effort to collect POA data (i.e., whether the hospital's relative performance would be materially impacted). Such a general approach could be applied to other types of enhanced administrative data (e.g., laboratory values, key clinical findings, etc.).

For discharge records with POA data the observed value of the data element for each covariate is used. For discharge records without POA data, the software calculates a predicted value for each covariate. For demographic and severity of illness covariates, the data elements are the same because these covariates are POA by definition. For comorbidity covariates, the software uses a 2x2 table of probabilities calculated on the discharges in the reference population with POA data. The four probabilities represent the following situations:

1. The covariate with POA is not present if the covariate ignoring POA[2] is not present

[2] Ignoring POA means that any secondary diagnosis code is considered both a comorbidity and a complication, depending on the context. For example, a secondary diagnosis code is considered a comorbidity when defining comorbiditeis, but a complication when defining the outcome of interest.

2. The covariate with POA is present if the covariate ignoring POA is not present
3. The covariate with POA is not present if the covariate ignoring POA is present
4. The covariate with POA is present if the covariate ignoring POA is present.

There is one 2x2 table per covariate. For discharge records without POA data, the predicted value for each comorbidity covariate is equal to the probability that the second or fourth situation above is true.

Using either the observed or predicted values for the covariates, the software calculates three predicted values for each discharge record. The first is the predicted value of the outcome of interest given the covariate values ignoring POA. The second is the predicted value of the outcome of interest given the covariate values using POA (either observed or predicted). The third is the predicted value of the data element that flags discharges for exclusion from the population at risk (denominator) given the covariate values using POA (either observed or predicted).

The expected rate for each hospital is an aggregate of the observed and predicted values for each discharge record in that hospital. At the hospital level, the software sums the number of flagged cases in the numerator and the number of flagged cases in the numerator that are either flagged as POA or predicted as POA. These two values are used to calculate the observed rate. The software then sums the number of predicted cases estimated by the risk-adjustment model module to yield the expected rate.

For additional background information and example calculations, see the document *Estimating Risk-Adjustment Models Incorporating Data on Present on Admission* on the AHRQ QI website (http://qualityindicators.ahrq.gov).

Risk-Adjusted Rate

The AHRQ QI use indirect standardization to calculate the risk-adjusted rate.

$$Risk\ adjusted\ Rate\ =\ (Observed\ Rate\ \div\ Expected\ Rate) \\ *\ Reference\ Population\ Rate$$

Note that for the reference population, the observed rate equals the expected rate equals the reference population rate equals the risk-adjusted rate.

Smoothed Rate

The formula for the smoothed rate is:

$$Smoothed\ Rate\ =\ (Risk\ Adjusted\ Rate \times\ Reliablity\ Weight) \\ +\ Reference\ Population\ Rate\ *\ (1 -\ Reliability\ Weight)$$

where

$$Reliability\ Weight\ = \frac{Signal\ Variance}{Signal\ Variance\ +\ Noise\ Variance}$$

Calculation of the confidence interval on the risk-adjusted rate and probability interval on the smoothed rate is discussed in the following sections.

Detailed Methods

The Analysis Module

The purpose of the Analysis Module (AM) is to fit a set of regression coefficients using the data of the reference population. The input dataset is expected to have variables corresponding to the outcome of interest at discharge Y, one or more indicators of an outcome of interest present on admission (POA indicators P), and covariate vectors $\boldsymbol{X}$ and $\boldsymbol{Z}$ containing demographic, condition, co-morbidity, and potentially any other information, used as explanatory variables. The covariate $\boldsymbol{X}$ is considered an improved measurement of the quantities measured by the covariate $\boldsymbol{Z}$**.** The outcome Y and covariate $\boldsymbol{Z}$ variables are never missing, but elements of the covariate $\boldsymbol{X}$ and values of the present-on-admission indicators P can be missing. The dataset also contains a hospital identification number and a record identification number (a key identifying unique discharge records.)

The purpose of the Prediction Module (PM) is to predict, for each discharge record, the expected value of the outcome of interest. These predictions are based on: i) an input dataset containing the same information, and having the same format as the analysis input dataset; and ii) a set of regression coefficients previously fitted by the Analysis Module using the data from the reference population. Since the outcome of interest is binary (either it is present or it is not), the expected value for each discharge can be viewed as the probability that the outcome of interest would have occurred for that discharge.

Missing Data

Missing data are handled by integrating the likelihood over all the possible values of the missing variables. This technique for dealing with missing data is well-established in the statistical literature. Little and Rubin [1] devote several chapters to analyzing missing data by integrating over the distribution, or likelihood, of the missing data. When the integral (or sum) of the likelihood cannot be feasibly calculated, an alternative method known as the Expectation-Maximization (EM) algorithm can be used. The EM algorithm was developed in the 1970s by Dempster, Laird and Rubin [2] to solve MLE equations in the presence of missing data. More recently, related methods based on Markov chain Monte Carlo (MCMC) algorithms have become popular for dealing with missing and censored data. MCMC algorithms include

methods such as Metropolis-Hastings or Gibbs sampling which are widely used in Bayesian statistical analysis [3]. MCMC methods are general and robust, and can be applied to a large variety of models. These methods are based on simulation, and they produce results that are approximations of the value being estimated. In particular, when POA data are missing, the results of the simulation, and therefore the hospital level expected rate that is estimated, may vary from one iteration of the software to the next—the variance is the "approximation error". The approximation error can be controlled by the number of MCMC steps used in the simulation. In particular, as the number of MCMC steps goes to infinity, the approximation error goes to zero. We will give detail about the MCMC used in the analysis and prediction module in the following sections.

Data Notation

Here is the general statistical notation used to describe the model:

- h_i is the hospital associated with the i^{th} record (patient);
- Y_i is a binary variable indicating the outcome of interest at hospital discharge associated with the i^{th} record. $Y_i = 1$ if the patient experiences the outcome of interest, $Y_i = 0$ otherwise;
- P_i is a binary variable indicating whether an outcome of interest is present on admission. Notice that if $Y_i = 0$, then it is assumed that $P_i = 0$. If more than one POA indicators are present, the maximum value is considered;
- $\boldsymbol{Z}_i$ is a vector of binary explanatory variables associated with the i^{th} record;
- $\boldsymbol{X}_i$ is a vector of improved binary explanatory variables associated with the i^{th} record.

In the following formulae i indicates the record index while k indicates the component index of the covariate vectors. For example, indicating with K the number of components of the covariate vectors, then $\boldsymbol{X}_i \in R^K$ indicates the vector of covariates associated with the i^{th} record, X_{ik} indicates the value of the k^{th} covariate associated with the i^{th} record, while X_k without the record index is used to indicate the k^{th} covariate of a generic covariate vector.

Statistical Model

The main goal of the model is the estimation of Y given $\boldsymbol{X}$ and $P = 0$. We assume the "conditional" binomial model

$$[Y|\boldsymbol{X}, P; \boldsymbol{\beta}_Y] = \prod_i \left(\pi_{Y,i}^{1-P_i}\right)^{Y_i} \left(1 - \pi_{Y,i}^{1-P_i}\right)^{1-Y_i} \quad (1)$$

with logistic link

$$\text{logit}\left(\pi_{Y,i}\right) = X_i \boldsymbol{\beta}_Y$$

Another component of the model is the estimation of P given $\boldsymbol{X}$, which is used to predict P when that values is missing. We assume the binomial model

$$[P|\boldsymbol{X};\boldsymbol{\beta}_P] = \prod_i \pi_{P,i}{}^{Y_i}(1-\pi_{P,i})^{1-Y_i} \tag{2}$$

with logistic link

$$\text{logit}(\pi_{P,i}) = X_i\boldsymbol{\beta}_P$$

Furthermore, we estimate $\boldsymbol{X}$ when element of that vector are missing by using the information contained in $\boldsymbol{Z}$. Since both $\boldsymbol{X}$ and $\boldsymbol{Z}$ contain binary variables, we model $[\boldsymbol{X}|\boldsymbol{Z}]$ using the two vectors of probabilities

$$\pi_{X,k}(0) = \Pr[X_k = 1|Z_k = 0]$$

$$\pi_{X,k}(1) = \Pr[X_k = 1|Z_k = 1]$$

and the likelihood

$$[\boldsymbol{X}|\boldsymbol{Z};\boldsymbol{\pi}_X] = \prod_{ik} \pi_{X,ik}^{X_{ik}}\left(1-\pi_{X,ik}\right)^{1-X_{ik}} \tag{3}$$

where

$$\pi_{X,\text{ik}} = \pi_{X,k}(Z_{ik})$$

Combining equations (1), (2) and (3), we obtain the likelihood

$$L(Y,\boldsymbol{X},P,\boldsymbol{Z};\boldsymbol{\beta}_Y,\boldsymbol{\beta}_X,\boldsymbol{\pi}_X) = [Y,\boldsymbol{X},P|\boldsymbol{Z};\boldsymbol{\beta}_Y,\boldsymbol{\beta}_X,\boldsymbol{\pi}_X] =$$

$$= [Y|\boldsymbol{X},P;\boldsymbol{\beta}_Y] \times [P|\boldsymbol{X};\boldsymbol{\beta}_P] \times [\boldsymbol{X}|\boldsymbol{Z};\boldsymbol{\pi}_X] =$$

$$= \prod_i \left(\pi_{Y,i}^{1-P_i}\right)^{Y_i}\left(1-\pi_{Y,i}^{1-P_i}\right)^{1-Y_i}\pi_{P,i}{}^{P_i}\left(1-\pi_{P,i}\right)^{1-P_i}\pi_{X,ik}^{X_{ik}}\left(1-\pi_{X,ik}\right)^{1-X_{ik}} \tag{4}$$

Likelihood (4) is written as a distribution of $Y,\boldsymbol{X},P$ given $\boldsymbol{Z}$. In order to write the model for missing $\boldsymbol{X}$ and P, we introduce the "imputed" variables $\boldsymbol{X'}$, $\boldsymbol{P'}$ and add the data model

$$\left[X_{ik}^{'}|X_{ik}\right] = \begin{cases} X_{ik} & X_{ik} \text{ is measured} \\ 1/2 & \text{otherwise} \end{cases} \tag{5}$$

$$\left[P_i^{'}|P_i\right] = \begin{cases} P_i & P_i \text{ is measured} \\ 1/2 & \text{otherwise} \end{cases} \tag{6}$$

The data model acts as a family of indicator variables, fixing the "imputed" variable to the measured value if the data are not missing. The likelihood integrated (summed) over the missing data can now be written as

$$\tilde{L}(Y, \boldsymbol{X}, P, \boldsymbol{Z}; \boldsymbol{\beta}_Y, \boldsymbol{\beta}_X, \boldsymbol{\pi}_X) = \sum_{P', X'} L(Y, \boldsymbol{X}', P, \boldsymbol{Z}'; \boldsymbol{\beta}_Y, \boldsymbol{\beta}_X, \boldsymbol{\pi}_X) \times [\boldsymbol{X}' | \boldsymbol{X}] \times [P' | P]$$

$$= \sum_{P', X'} [Y | \boldsymbol{X}, P; \boldsymbol{\beta}_Y] \times [P | \boldsymbol{X}; \boldsymbol{\beta}_P] \times [\boldsymbol{X} | \boldsymbol{Z}; \boldsymbol{\pi}_X] \times [\boldsymbol{X}' | \boldsymbol{X}] \times [P' | P] \quad (7)$$

As the number of components of the covariate vector $\boldsymbol{X}$ increases, it becomes unfeasible to compute the above sum deterministically. For example, if $\boldsymbol{X}$ has 30 components, then the number of sums for every record with missing $\boldsymbol{X}$ data is $2^{30} > 10^9$, and if the number of components is 100, then the number of sums becomes $2^{100} > 10^{30}$. The AM and PM employ alternative methods for integrating (summing) the likelihood over the missing data.

Model Fitting Approach using MCMC

To fit the $\boldsymbol{\beta}_Y$ coefficients using the *marginal* likelihood (7) (that is, the likelihood integrated over the missing data), we use Metropolis-Hastings and Gibbs sampling algorithms, which are standard MCMC techniques (see [3]).

After reading the data, the AM fits the coefficients $\widehat{\boldsymbol{\beta}}_P$ and $\widehat{\boldsymbol{\pi}}_X$ using only the records in the dataset that have no missing data. Then, given $\widehat{\boldsymbol{\beta}}_P$ and $\widehat{\boldsymbol{\pi}}_X$, a sample of values of $\boldsymbol{\beta}_Y$, $\boldsymbol{X'}$, and P' is drawn from the posterior distribution:

$$\left[\boldsymbol{X}', P', \boldsymbol{\beta}_Y | Y, \boldsymbol{X}, P, \boldsymbol{Z}; \widehat{\boldsymbol{\beta}}_X, \widehat{\boldsymbol{\pi}}_X\right] \propto [Y | \boldsymbol{X}, P; \boldsymbol{\beta}_Y] \times \left[P \middle| \boldsymbol{X}; \widehat{\boldsymbol{\beta}}_P\right] \times [\boldsymbol{X} | \boldsymbol{Z}; \widehat{\boldsymbol{\pi}}_X] \times [\boldsymbol{X}' | \boldsymbol{X}] \times [P' | P]$$

Metropolis-Hastings is used to sample $\boldsymbol{X'}$ and P', and Gibbs sampling is used to sample $\boldsymbol{\beta}_Y$. The sampling equations are the following:

- Sampling of P' (Metropolis-Hastings)

$$P'_{new} \sim [P' | P]$$

$$acceptance = \min\left(\frac{[Y | \boldsymbol{X}, P_{new}; \boldsymbol{\beta}_Y] \times \left[P_{new} \middle| \boldsymbol{X}; \widehat{\boldsymbol{\beta}}_P\right] \times [\boldsymbol{X}' | \boldsymbol{X}]}{[Y | \boldsymbol{X}, P; \boldsymbol{\beta}_Y] \times \left[P \middle| \boldsymbol{X}; \widehat{\boldsymbol{\beta}}_P\right] \times [\boldsymbol{X}' | \boldsymbol{X}]}, 1\right)$$

- Sampling of $\boldsymbol{X'}$ (Metropolis-Hastings)

$$\boldsymbol{X}'_{new} \sim [\boldsymbol{X}' | \boldsymbol{X}]$$

$$acceptance = \min\left(\frac{[Y | \boldsymbol{X}_{new}, P; \boldsymbol{\beta}_Y] \times [\boldsymbol{X}_{new} | \boldsymbol{Z}; \widehat{\boldsymbol{\pi}}_X] \times [P' | P]}{[Y | \boldsymbol{X}, P; \boldsymbol{\beta}_Y] \times [\boldsymbol{X} | \boldsymbol{Z}; \widehat{\boldsymbol{\pi}}_X] \times [P' | P]}, 1\right)$$

- Sampling of $\boldsymbol{\beta}_Y$ (Gibbs sampling)

$$\boldsymbol{\beta}_{Y,new} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \times N(\mathbf{0}, \sigma^2 \mathbf{I})$$

$$acceptance = 1$$

where $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ is the multivariate normal approximation of the function

$$\boldsymbol{\beta}_Y \rightarrow [Y|\boldsymbol{X}, P; \boldsymbol{\beta}_Y] = \prod_i \left(\pi_{Y,i}^{1-P_i}\right)^{Y_i} \left(1 - \pi_{Y,i}^{1-P_i}\right)^{1-Y_i}$$

The AM includes an option to use Generalized Estimating Equations ([4], [5], [6]) with an exchangeable correlation model to account for within hospital h_i correlation. The normal distribution $N(\mathbf{0}, \sigma^2\mathbf{I})$ represents a non-informative prior distribution (for small values of the precision $\tau = 1/\sigma^2$) added to regularize cases with separable data.

Analysis Module Output

In addition to the quantities $\widehat{\boldsymbol{\beta}}_Y$, $\widehat{\boldsymbol{\beta}}_P$, $\widehat{\boldsymbol{\pi}}_X$ discussed above, the analysis module also calculates, for comparison purposes, the regression coefficients of the binomial model $[Y|\boldsymbol{Z}]$ fitted using all the data, the binomial model $[Y|\boldsymbol{X}]$ fitted using all the non-missing data, and the binomial model $[Y|\boldsymbol{X}, P = 0]$ fitted using all the non-missing data with $P = 0$.

References:

1. Little, R.J.A. and Rubin, D.B. (2002). *Statistical Analysis with Missing Data*. Wiley, Hoboken, NJ.
2. Dempster, A.P.; Laird, N.M.; Rubin, D.B. (1977). Maximum Likelihood from Incomplete Data via the EM Algorithm. *Journal of the Royal Statistical Society, Series B (Methodological)* 39 (1): 1–38.
3. Robert, C.P. and Casella, G. (2004). *Monte Carlo Statistical Methods*, 2nd Ed. Springer, New York, NY.
4. Scott L. Zeger and Kung-Yee Liang. Longitudinal Data Analysis for Discrete and Continuous Outcomes. *Biometrics*, 42(1):121-130, March 1986.
5. Kung-Yee Liang and Scott L. Zeger. Longitudinal Data Analysis Using Generalized Linear Models. *Biometrika*, 73(1):13-22, April 1986
6. Garrett M. Fitzmaurice, Nan M. Laird, and James H. Ware. *Applied Longitudinal Analysis*. John Wiley & Sons, Inc., thirteenth edition, 2004.

The Prediction Module

The purpose of the Prediction Module (PM) is to predict, for each discharge record, the expected value of the adverse health outcome. These predictions are based on: i) an input dataset containing the same information, and having the same format as the analysis input dataset; and ii) a set of regression coefficients previously fitted by the Analysis Module using the data from a reference population. Since the adverse health outcome is binary (either it is present or it is not), the expected value for each discharge can be viewed as the probability that the adverse health outcome would have occurred for that discharge. These calculations are straightforward when there are no missing data, but they require high dimensional sums when data are missing. Let $\widehat{\boldsymbol{\beta}}_Y$, $\widehat{\boldsymbol{\beta}}_P$, $\widehat{\boldsymbol{\pi}}_X$ be the regression coefficients fit by the AM as described in the previous section, and set

$$p(\boldsymbol{X}', P') := [\boldsymbol{X}', P' | Y, \boldsymbol{X}', P, \boldsymbol{Z}'; \boldsymbol{\beta}_Y, \boldsymbol{\beta}_X, \boldsymbol{\pi}_X] \propto$$

$$\propto [Y|\boldsymbol{X}, P; \boldsymbol{\beta}_Y] \times \left[P\middle|\boldsymbol{X}; \widehat{\boldsymbol{\beta}}_P\right] \times [\boldsymbol{X}|\boldsymbol{Z}; \widehat{\boldsymbol{\pi}}_X] \times [\boldsymbol{X}'|\boldsymbol{X}] \times [P'|P]$$

The main goal of the prediction module is to calculate

$$\Pr[Y_i = 1|\boldsymbol{X}_i, P_i = 0]$$

where we explicitly use the index i to indicate that the prediction is performed at the discharge record. For a record where both P_i and $\boldsymbol{X}_i$ are measured and $P_i = 0$, the predicted probability is simply given by

$$\Pr[Y_i = 1|\boldsymbol{X}_i, P_i = 0] = \hat{\pi}_Y(\boldsymbol{X}_i) \equiv \text{logit}^{-1}\left(\boldsymbol{X}_i \widehat{\boldsymbol{\beta}}_Y\right)$$

If P_i is missing, then we calculate the expected value of $\hat{\pi}_Y(\boldsymbol{X}_i)\left(1 - P_i'\right)$ over the distribution of the missing data $p\left(\boldsymbol{X}_i', P_i'\right)$, namely

$$\sum_{P_i'=\{0,1\}} \hat{\pi}_Y(\boldsymbol{X}_i)\left(1 - P_i'\right) p\left(\boldsymbol{X}_i', P_i'\right) = \hat{\pi}_Y(\boldsymbol{X}_i) p(\boldsymbol{X}_i, 0) \equiv \text{logit}^{-1}\left(\boldsymbol{X}_i \widehat{\boldsymbol{\beta}}_Y\right) p(\boldsymbol{X}_i, 0)$$

which is quick to compute. The general case however, where P_i and/or any combination of components of the vector $\boldsymbol{X}_i$ is missing, requires the sum over all the possible combinations of missing values:

$$\widehat{\Pi}_i = \sum_{P_i', \boldsymbol{X}_i'} \hat{\pi}_Y(\boldsymbol{X}_i)\left(1 - P_i'\right) p\left(\boldsymbol{X}_i', P_i'\right) =$$

$$= \sum_{P_i', \boldsymbol{X}_i'} \text{logit}^{-1}\left(\boldsymbol{X}_i' \widehat{\boldsymbol{\beta}}_Y\right)\left(1 - P_i'\right) p\left(\boldsymbol{X}_i', P_i'\right) \quad (8)$$

where the capitol $\widehat{\Pi}_i$ used in (8) shall not be confused with the product operator. Following the same argument used in the previous section, as the number of components of the vector of covariate $\boldsymbol{X}$ increases, the deterministic sum quickly becomes unfeasible and an alternative approach is necessary. In this case, we evaluated the multidimensional sum using a Metropolis-Hasting implementation of the Importance Sampling Monte Carlo integration method (see chapter 7, paragraphs 7.6, 7.7 of the celebrated Numerical Recipes book [7] for a primer introduction on Monte Carlo integration, references [8], [9], [10] for a deeper discussion, or many of the papers on the subject that can be freely found online.)

The methods works as follows: we draw a sample of imputed $\boldsymbol{X}_i', P_i'$ values from the distribution $p\left(\boldsymbol{X}_i', P_i'\right)$, namely

$$\left(\boldsymbol{X}_{i,s}', P_{i,s}'\right) \sim\ p\left(\boldsymbol{X}_i', P_i'\right) \quad s = 1, \ldots, N$$

using the Metropolis-Hastings to sample $\boldsymbol{X}_i^{'}$ and $P_i^{'}$ discussed in the Analysis Module section, then we approximate the sum (8) with the sample sum

$$I_N = \frac{1}{N}\sum_{s=1}^{N} \hat{\pi}_Y\left(\boldsymbol{X}_{i,s}^{'}\right)\left(1 - P_{i,s}^{'}\right)$$

Because the Metropolis-Hastings algorithm samples from p by generating a Markov chain, this method can be considered a MCMC method.

The numerical approximation of the Monte Carlo integration is known to be controlled by the sample variance

$$V_N = \frac{1}{N-1}\sum_{s=1}^{N}\left(\hat{\pi}_Y\left(\boldsymbol{X}_{i,s}^{'}\right)\left(1 - P_{i,s}^{'}\right)\right)^2 - \frac{N}{N-1}\left(\frac{1}{N}\sum_{s=1}^{N}\hat{\pi}_Y\left(\boldsymbol{X}_{i,s}^{'}\right)\left(1 - P_{i,s}^{'}\right)\right)^2$$

Since the distribution p has compact support and the function $\hat{\pi}_Y(\boldsymbol{X}_i)$ is bounded, then the variance V_N is also bounded. Therefore, under the assumption that the sample $\left(\boldsymbol{X}_{i,s}^{'}, P_{i,s}^{'}\right)$ is ergodic (i.e. random), it follows from the central limit theorem that

$$I_N \to \hat{\Pi}_i$$

in a probabilistic sense with a standard error equal to

$$\sigma_N = \sqrt{V_N/N}$$

The value V_N can be calculated together with I_N to provide an estimate of the Monte Carlo approximation error. However, regardless of V_N, the error of the MCMC integration scales as $1/\sqrt{N}$.

The PM also calculates, for comparative purposes, the expected values of the predictor $\hat{\pi}_Y$ for the different sets of coefficients $\widehat{\boldsymbol{\beta}}_Y$ estimated in the Analysis Module, the expected values of the predictor $\hat{\pi}_P$, and the marginal probability of $P_i^{'} = 1$ given by

$$\sum_{\boldsymbol{X}_i^{'}} p\left(\boldsymbol{X}_i^{'}, 1\right)$$

References:

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Computing the Risk-Adjusted Rate Variance

Let

- E_i be the predicted rate (8);
- n_h be the number of discharges at hospital h; and
- α be the reference population rate (average outcome in the entire sample).

We define the observed rate at hospital h as

$$O_h = \frac{1}{n_h} \sum_{\substack{i \\ h_i = h}} Y_i$$

the expected rate at hospital h as

$$E_h = \frac{1}{n_h} \sum_{\substack{i \\ h_i = h}} \widehat{\Pi}_i$$

and the Risk Adjusted Rate

$$RAR_h = \alpha \times \frac{O_h}{E_h}$$

Using a Taylor expansion for the formula for the variance of the ratio of two stochastic variables R, S

$$Var\left(\frac{R}{S}\right) \cong \frac{E[R]^2}{E[R]^2}\left(\frac{Var(R)}{E[R]^2} - 2\frac{Cov(R,S)}{E[R]E[S]} + \frac{Var(S)}{E[S]^2}\right)$$

we compute the variance on the risk-adjusted rate

$$Var(RAR_h) \cong \alpha^2 \frac{E[O_h]^2}{{E_h}^2}\left(\frac{Var(O_h)}{E[O_h]^2} - 2\frac{Cov(O_h, E_h)}{E[O_h]E_h} + \frac{Var(E_h)}{{E_h}^2}\right)$$

It is common practice in these calculations to neglect the variance of the predictor $\widehat{\boldsymbol{\Pi}}_{\boldsymbol{i}}$ (see [11]) and to consider a normal distribution for the Risk Adjusted Rate (only true in the limit $\boldsymbol{n_h} \to \infty$). In this case the above formula simplifies to

$$Var(RAR_h) \cong \alpha^2 \frac{Var(O_h)}{{E_h}^2}$$

and the 95% confidence intervals are calculated assuming normality. However, arguments to support using non-approximate equations (see [12] for an example) for the ***RAR*** confidence intervals (in particular when $\boldsymbol{n_h}$ is small) may be considered in future releases of the AHRQ QI software.

References:

11. David W. Hosmer, Stanley Lemeshow (1995). Confidence interval estimates of an index of quality performance based on logistic regression. Statistics in Medicine, Vol 14, Issue 19, 2161-2172
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Computing the Smoothed Rate Variance

The detailed formula for calculating the probability interval around the smoothed rate is described in the section below on composite measures. Calculation of the smoothed rate is a step in the process of computing the composite measures. However, the basic formula is:

$$\begin{aligned} Smoothed\ Rate\ &= (Risk\ Adjusted\ Rate \times\ Reliablity\ Weight) \\ &+ Reference\ Population\ Rate\ *\ (1 - Reliability\ Weight) \end{aligned}$$

$$Reliability\ Weight\ = \frac{Signal\ Variance}{Signal\ Variance\ +\ Noise\ Variance}$$

$$Posterior\ Variance\ =\ Signal\ Variance - (Reliability\ Weight\ *\ Signal\ Variance)$$

The $smoothed\ rate$ to follows a Gamma distribution $G(shape, scale)$ where

$$shape\ = \frac{(Smoothed\ Rate)^2}{Posterior\ Variance}$$

$$scale\ = \frac{Posterior\ Variance}{Smoothed\ Rate}$$

A 95% probability interval can be calculated using the inverse CDF of the gamma distribution as

$$\begin{aligned} lower\ bound\ &= inv\_cdf\_gamma(0.025, shape, scale) \\ upper\ bound\ &= inv\_cdf\_gamma(0.975, shape, scale) \end{aligned}$$

Composite Measures

Overview

The general methodology for the AHRQ QI composite measures might be described as constructing a "composite of composites." The first "composite" is the reliability-adjusted ratio, which is a weighted average of the risk-adjusted ratio and the reference population ratio, where the weight is determined empirically. The second "composite" is a weighted average of the component indicators, where the weights are selected based on the intended use of the composite measure. These weights might be determined empirically or based on non-empirical considerations.

Composite Value

The basic steps for computing the composite are as follows:

Step 1. Compute the risk-adjusted rate and confidence interval

The AHRQ QI risk-adjusted rate and confidence interval are computed as described above.

Step 2. Scale the risk-adjusted rate using the reference population

The levels of the rates vary from indicator to indicator. To combine the component indicators using a common scale, each indicator's risk-adjusted rate is first divided by the reference population rate to yield a ratio. The components of the composite are therefore defined in terms of a ratio to the reference population rate for each indicator. The component indicators are scaled by the reference population rate so that each indicator reflects the degree of deviation from the overall average performance.

Step 3. Compute the reliability-adjusted ratio

The reliability-adjusted ratio is computed as the weighted average of the risk-adjusted ratio and the reference population ratio, where the weights vary from 0 to 1, depending on the degree of reliability for the indicator and provider (or other unit of analysis).

$$Reliability\ Adjusted\ Ratio\ = (risk - adjusted\ ratio\ \times\ weight) + reference\ population\ ratio\ \times\ (1 - weight)$$

For small providers, the weight is closer to 0. For large providers, the weight is closer to 1. For a given provider, if the denominator is 0, then the weight assigned is 0 (i.e., the reliability-adjusted ratio is the reference population ratio).

Step 4. Select the component weights

The composite measure is the weighted average of the scaled and reliability-adjusted ratios for the component indicators.

Single indicator weight. In this case, the composite is simply the reliability-adjusted ratio for a single indicator. The reference population rate is the same among all providers.

Equal weight. In this case, each component indicator is assigned an identical weight based on the number of indicators. That is, the weight equals 1 divided by the number of indicators in the composite (e.g., 1/11 = 0.0909).

Numerator weight. A numerator weight is based on the relative frequency of the numerator for each component indicator in the reference population. In general, a numerator weight reflects the amount of harm in the outcome of interest, in this case a potentially preventable adverse event. One might also use weights that reflect the amount of excess mortality or complications associated with the adverse event, or the amount of confidence one has in identifying events (i.e., the positive predictive value).

Denominator weight. A denominator weight is based on the relative frequency of the denominator for each component indicator in the reference population. In general, a denominator weight reflects the degree of risk of experiencing the outcome of interest in a given population. For example, the denominator weight might be based on the demographic composition of a health plan, the employees of a purchaser, a state, an individual hospital, or a single patient.

Factor weight. A factor weight is based on some sort of analysis that assigns each component indicator a weight that reflects the contribution of that indicator to the common variation among the indicators. The component indicator that is most predictive of that common variation is assigned the highest weight. The weights for each composite are based on a principal components factor analysis of the reliability-adjusted ratios.

Step 5. Construct the composite measure

The composite measure is the weighted average of the component indicators using the selected weights and the scaled and reliability-adjusted indicators.

$$Composite = (indicator_1\ RAR \times weight_1) + (indicator_2\ RAR \times weight_2) + \cdots + (indicator_N\ RAR \times weight_N)$$

Composite Variance

The probability interval of the composite measure is based on its standard error, which is the square root of the variance. The variance is computed based on the signal variance-covariance matrix and the reliability weights.

Let M be a $1 \times K$ vector of observed quality measures (for a given hospital, suppress hospital subscript for convenience), noisy measures of the true underlying $1 \times K$ quality vector μ, such that:

$$M = \mu + \varepsilon \tag{9}$$

where ϵ is a $1 \times K$ noise vector with zero mean and $K \times K$ variance-covariance matrix $Var(\epsilon) = \Omega_\epsilon$. Let the $K \times K$ signal variance-covariance be $Var(\mu) = \Omega_\mu$.

Let $\hat{\mu}$ a $1 \times K$ vector indicating the posterior (filtered) estimate of μ, such that:

$$\hat{\mu} = \mu + \nu \tag{10}$$

where ν is a $1 \times K$ vector with zero mean and $K \times K$ variance-covariance matrix $Var(\nu)$ representing the prediction error of the posterior estimates.
The goal is to estimate the variance for any weighted average of the posterior estimates. For a given $1 \times K$ weighting vector w, this is given by:

$$Var(\nu w) = w' Var(\nu) w$$

where w' indicates the transpose of w.

Thus, we need an estimate of $Var(\nu)$. We simplify the calculation by assuming that the posterior (filtered) estimates are formed in isolation for each measure (i.e. univariate) and the estimation error is assumed not correlated across measures (e.g., each measure is based on a different sample of patients or independent patient outcomes).

Forming each measure in isolation, using superscripts $k = 1, \ldots, K$ to indicate the measure, we have:

$$\hat{\mu}^k = M^k \hat{\beta}^k = M^k \left(\Omega_\mu^{kk} + \Omega_\epsilon^{kk}\right)^{-1} \Omega_\mu^{kk} \tag{11}$$

$$Var(\nu^k) = \Omega_\mu^{kk}\left(1 - \hat{\beta}^k\right) = \Omega_\mu^{kk} - \Omega_\mu^{kk}\left(\Omega_\mu^{kk} + \Omega_\epsilon^{kk}\right)^{-1} \Omega_\mu^{kk} \tag{12}$$

where

$$\hat{\beta}^k = \left(\Omega_\mu^{kk} + \Omega_\epsilon^{kk}\right)^{-1} \Omega_\mu^{kk}$$

is the signal ratio of measure k, the reliability of the measure, and is the r-squared which measures how much of the variation in the true measure can be explained with the filtered measure. Note that in this simplified case the filtered estimate is a univariate shrinkage estimator.

For the non-diagonal elements of the covariance matrix (for $j \neq k$):

$$Cov\left(\nu^j, \nu^k\right) = E\left[\left(\mu^j - \hat{\mu}^j\right)\left(\mu^k - \hat{\mu}^k\right)\right] \tag{13}$$

assuming independent estimation error in the two measures, one gets the following simplified expression (see supplemental notes below for the derivation):

$$Cov(v^j, v^k) = \Omega_\mu^{jk}(1-\hat{\beta}^j)(1-\hat{\beta}^k) \quad (14)$$

Note that this is just the signal covariance times 1 minus the signal ratio for each of the measures. Thus, if the signal ratio is 0 for each measure, the covariance in the estimates is simply the signal covariance. As either measure gets a stronger signal ratio (becomes more precise), the covariance in the estimates shrinks to 0.

Also note that if one measure is missing, then the signal ratio is simply set to 0. The filtered estimate is shrunk all the way back to the (conditional) mean, and the variance and covariance are as defined above.

The standard error on the composite is the square root of the variance, which is then used to compute the 95% probability interval.

The $composite\ value$ follows a Gamma distribution $G(shape, scale)$ where

$$shape = \frac{(Composite\ Value)^2}{Posterior\ Variance}$$

$$scale = \frac{Posterior\ Variance}{Composite\ Value}$$

A 95% probability interval can be calculated using the inverse CDF of the gamma distribution as

$$lower\ bound = inv\_cdf\_gamma(0.025, shape, scale)$$
$$upper\ bound = inv\_cdf\_gamma(0.975, shape, scale)$$

References:

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Supplemental Notes:

To derive formula (14), we substitute

$$\hat{\mu} = M\hat{\beta} = (\mu + \epsilon)\hat{\beta}$$

into (13) and obtain (for $j \neq k$)

$$Cov(v^j, v^k) = E\left[\left(\mu^j - \left(\mu^j + \epsilon^j\right)\hat{\beta}^j\right)\left(\mu^k - (\mu^k + \epsilon^k)\hat{\beta}^k\right)\right] =$$

$$= E\left[\left(\mu^j(1 - \hat{\beta}^j) - \epsilon^j\hat{\beta}^j\right)\left(\mu^k(1 - \hat{\beta}^k) - \epsilon^k\hat{\beta}^k\right)\right] =$$

$$= E\left[\mu^j\mu^k\left(1 - \hat{\beta}^j\right)\left(1 - \hat{\beta}^k\right) + \mu^k\epsilon^j\left(1 - \hat{\beta}^k\right)\hat{\beta}^j + \mu^j\epsilon^k\left(1 - \hat{\beta}^j\right)\hat{\beta}^k + \epsilon^j\epsilon^k\hat{\beta}^j\hat{\beta}^k\right] =$$

$$= E\left[\mu^j\mu^k\right]\left(1 - \hat{\beta}^j\right)\left(1 - \hat{\beta}^k\right) + E\left[\mu^k\epsilon^j\right]\left(1 - \hat{\beta}^k\right)\hat{\beta}^j + E\left[\mu^j\epsilon^k\right]\left(1 - \hat{\beta}^j\right)\hat{\beta}^k + E\left[\epsilon^j\epsilon^k\right]\hat{\beta}^j\hat{\beta}^k$$

Assuming $E\left[\mu^j\epsilon^k\right] = E\left[\epsilon^j\mu^k\right] = E\left[\epsilon^j\epsilon^k\right] = 0$ and $E[\mu] = 0$, we have

$$Cov(v^j, v^k) = E\left[\mu^j\mu^k\right]\left(1 - \hat{\beta}^j\right)\left(1 - \hat{\beta}^k\right) =$$

$$= Cov\left(\mu^j, \mu^k\right)\left(1 - \hat{\beta}^j\right)\left(1 - \hat{\beta}^k\right) - E\left[\mu^j\right]E[\mu^k]\left(1 - \hat{\beta}^j\right)\left(1 - \hat{\beta}^k\right) =$$

$$= Cov\left(\mu^j, \mu^k\right)\left(1 - \hat{\beta}^j\right)\left(1 - \hat{\beta}^k\right).$$

QED

Appendix A. Table of AHRQ QI Risk Adjustment / POA

Appendix Table A.1 denotes which AHRQ QI are risk-adjusted and which use POA data and for what purpose (i.e., for technical specifications or risk adjustment).

Appendix Table A.1. AHRQ QI Risk Adjustment and Uses of POA

		Use POA?	
	Calculate Risk Adjusted Rate	Technical Specifications	Risk Adjustment
IQI #01 - Esophageal Resection Volume			
IQI #02 - Pancreatic Resection Volume			
IQI #04 - AAA Repair Volume			
IQI #05 - CABG Volume			
IQI #06 - PTCA Volume			
IQI #07 - Carotid Endarterectomy Volume			
IQI #08 - Esophageal Resection Mortality	X		X
IQI #09 - Pancreatic Resection Mortality	X		X
IQI #11 - AAA Repair Mortality	X		X
IQI #12 - CABG Mortality	X		X
IQI #13 - Craniotomy Mortality	X		X
IQI #14 - Hip Replacement Mortality	X		X
IQI #15 - AMI Mortality	X		X
IQI #16 - CHF Mortality	X		X
IQI #17 - Acute Stroke Mortality	X		X
IQI #18 - GI Hemorrhage Mortality	X		X
IQI #19 - Hip Fracture Mortality	X		X
IQI #20 - Pneumonia Mortality	X		X
IQI #21 - cesarean section delivery			

		Use POA?	
	Calculate Risk Adjusted Rate	Technical Specifications	Risk Adjustment
IQI #22 - Vaginal birth after C-section, uncomplicated			
IQI #23 - Laparoscopic cholecystectomy			
IQI #24 - Incidental appendectomy			
IQI #25 - Bi-lateral catheterization			
IQI #26 - Coronary artery bypass graft	X		
IQI #27 - PTCA	X		
IQI #28 – Hysterectomy	X		
IQI #29 - Laminectomy and/or spinal fusion	X		
IQI #30 - PTCA Mortality	X		X
IQI #31 - Carotid Endarterectomy Mortality	X		X
IQI #32 – AMI Mortality WO Transfer	X		X
IQI #33 - Primary cesarean section			
IQI #34 - VBAC, all			
PSI #02 – Death in Low Mortality DRGs	X		X
PSI #03 - Pressure Ulcer	X	X	X
PSI #04 - Death among Surgical In-patients with Serious Treatable Complications	X		X
PSI #05 - Foreign Body left in During Procedure		X	
PSI #06 - Iatrogenic Pneumothorax	X	X	X
PSI #07 - Central Venous Catheter-related BSI	X	X	X
PSI #08 - Post-operative Hip Fracture	X	X	X
PSI #09 - Post- operative Hemorrhage or Hematoma	X	X	X
PSI #10 - Post- operative Physiologic & Metabolic Derangement	X	X	X
PSI #11 - Post- operative Respiratory Failure	X	X	X
PSI #12 - Post- operative PE or DVT	X	X	X
PSI #13 - Post- operative Sepsis	X	X	X

		Use POA?	
	Calculate Risk Adjusted Rate	Technical Specifications	Risk Adjustment
PSI #14 - Post- operative Wound Dehiscence	X		X
PSI #15 - Accidental Puncture or Laceration	X	X	X
PSI #16 - Transfusion Reaction		X	
PSI #17 - Birth Trauma - Injury to Neonate			
PSI #18 - OB Trauma – Vaginal with Instrument-assisted Delivery			
PSI #19 - OB Trauma – Vaginal without Instrument-assisted Delivery			
PDI #01 - Accidental Puncture or Laceration	X	X	X
PDI #02 - Pressure Ulcer	X	X	X
PDI #03 - Foreign Body left in During Procedure		X	
PDI #05 - Iatrogenic Pneumothorax	X	X	X
PDI #06 - Pediatric Heart Surgery Mortality	X		X
PDI #07 - Pediatric Heart Surgery Volume			
PDI #08 - Post- operative Hemorrhage or Hematoma	X	X	X
PDI #09 - Post- operative Respiratory Failure	X	X	X
PDI #10 - Post- operative Sepsis	X	X	X
PDI #11 - Post- operative Wound Dehiscence			X
PDI #12 - Central Venous Catheter-related BSI	X	X	X
PDI #13 - Transfusion Reaction		X	
NQI #01 - Iatrogenic Pneumothorax in Neonates		X	X
NQI #02 - Neonatal Mortality	X		X
NQI #03 - Blood Stream Infections in Neonates	X	X	X
PQI #01 - Diabetes short-term complications	X		
PQI #02 - Perforated appendix	X		
PQI #03 - Diabetes long-term complications	X		
PQI #05 – COPD or asthma in older adults	X		

		Use POA?	
	Calculate Risk Adjusted Rate	Technical Specifications	Risk Adjustment
PQI #07 - Hypertension	X		
PQI #08 - Congestive heart failure	X		
PQI #10 - Dehydration	X		
PQI #11 - Bacterial pneumonia	X		
PQI #12 - Urinary infections	X		
PQI #13 - Angina without procedure	X		
PQI #14 - Uncontrolled diabetes	X		
PQI #15 – Asthma in younger adults	X		
PQI #16 - Lower extremity amputation among patients with diabetes	X		

IQI = Inpatient Quality Indicator; PSI = Patient Safety Indicator; PDI = Pediatric Quality Indicator; NQI = Neonatal Quality Indicator

Appendix B. Table of AHRQ QI Risk Adjustment Covariates

The categories highlighted in blue are mutually exclusive and exhaustive, meaning that every discharge is assigned a value of "1" for one and only one covariate and there must be an omitted covariate (usually the most common or the least risk). If covariates within a highlighted category are excluded because N<30 or p<0.05 then the covariate is combined with another along the risk gradient. For example, combine birth weight 500-999g with 1000-1499g, age 18-24 with age 25-29 or combine ROM subclass "4" with ROM subclass "3".

Appendix Table B.1 Table of AHRQ QI Risk Adjustment Covariates for Provider Level Indicators

Category	Mutually Exclusive	IQI	PSI	PDI	NQI
Demographics		Sex	Sex	Sex	Sex
		Age (5-year age groups)	Age (5-year age groups)	Birth weight (500g groups) Age in days (90 days to 1 year) Age in years (1 year and above)	Birth weight (500g groups)
Severity of Illness	DRGs pool into MDCs	APR-DRG Major Diagnosis Categories (MDC)	Modified MS-DRG* Major Diagnosis Categories (MDC)	Modified MS-DRG* Major Diagnosis Categories (MDC)	Modified MS-DRG* Major Diagnosis Categories (MDC)
Comorbidities		APR-DRG Risk of mortality subclass (1 – minor; 2 - moderate; 3 – major; 4 – extreme)	AHRQ Comorbidities	AHRQ Clinical Classification Software	Congenital anomalies
Other		Transfer-in status Point of Origin status	Transfer-in status Point of Origin status Days to Procedure status	Transfer-in status Point of Origin status Days to Procedure status Indicator-specific risk stratifiers	Transfer-in status Point of Origin status Days to Procedure status

* Prior to October 1, 2007 use CMS-DRGs; highlighted categories are mutually exclusive with an omitted covariate.